Commun. Combin., Cryptogr. & Computer Sci., 1 (2025), 50–59

ISSN:2783-5456

Communications in Combinatorics, Cryptography & Computer Science

Journal Homepage: http://cccs.sgh.ac.ir

A Data-Driven Framework for State-Feedback Control via Mapping in an LMI Structure to Satisfy the Generalized Lyapunov Condition

Hossein valikelari *[a], Ahmadreza vali *[a], Abdorreza Kashaninia[a]

[a]Faculty of Electrical & Computer Engineering, Malek-Ashtar University of Technology, Tehran, Iran.

Abstract

This paper proposes a novel data-driven robust control framework for uncertain nonlinear systems, integrating system identification, spectral analysis, and sliding mode control (SMC). Initially, the dynamic model is extracted directly from input-output data using the Dynamic Mode Decomposition with Control (DMDc) algorithm. To ensure model fidelity, the identified structure is rigorously evaluated via spectral analysis, focusing on eigenvalues and singular values to isolate dominant modes and effective dynamic components. The control gains are derived by solving an optimization problem governed by a cost function and the discrete Riccati equation, which serves as the foundation for the SMC law design. Furthermore, operational constraints concerning the magnitude of control effort and the rate of change of the control signal are explicitly incorporated into the design process to ensure robust and reliable performance.

Keywords: Data-Driven Control, Dynamic Mode Decomposition with Control (DMDc), Sliding Mode Control, Discrete Riccati Equation.

1. Introduction

In recent decades, controlling uncertain nonlinear systems has remained a fundamental challenge in control engineering due to their complex dynamics, unknown parameters, and the difficulty of obtaining accurate mathematical models [1]. Although physics-based modeling approaches provide reliable representations of system behavior, they often require extensive prior knowledge and may suffer from high computational complexity when applied to complex nonlinear systems [2]. Therefore, data-driven methodologies have emerged as effective alternatives by extracting dynamic representations directly from measured data without requiring complete prior knowledge of system structures [3]. Among these approaches, Dynamic Mode Decomposition with Control (DMDc) has attracted significant attention as a data-driven identification technique for constructing linear input-output models of dynamical systems and enabling control-oriented modeling [4]. However, ensuring robustness against uncertainties, external disturbances, and unmodeled dynamics remains a critical issue; therefore, combining data-driven models with robust control approaches such as Sliding Mode Control (SMC) has become an important research direction [5].

In recent years, the transition from conventional model-based control strategies toward model-free and data-driven control frameworks has received considerable attention due to their improved adaptability in uncertain and varying operating conditions [6]. In addition to performance requirements, guaranteeing

*Corresponding author
Email addresses: Hossein.valikelaeriu@gmail.com (Hossein valikelari *), akashaninia@mut.ac.ir (Ahmadreza vali *)

Received: July 5, 2026 Revised: August 3, 2026 Accepted: August 5, 2026

system safety has become an essential consideration, where Control Barrier Functions (CBFs) provide a systematic framework for enforcing safety constraints during controller operation [7]. Meanwhile, several advanced sliding mode control methodologies have been developed to enhance robustness and reduce the undesirable chattering effect associated with conventional SMC designs [8]. Furthermore, optimal control techniques based on Linear Quadratic Regulator (LQR) theory and the discrete-time Riccati equation provide a rigorous mathematical framework for computing optimal feedback gains while balancing control effort and system performance [9].

Beyond traditional identification methods, spectral analysis of the Koopman operator has introduced a powerful perspective for representing nonlinear dynamical systems through infinite-dimensional linear operators, providing valuable insights into complex system behaviors [10]. Building upon this concept, Koopman-based predictive models combined with Model Predictive Control (MPC) have demonstrated promising capabilities for controlling nonlinear systems using data-driven linear representations [11]. Similarly, DMDc-based controller design approaches have shown effectiveness in constructing control strategies for nonlinear dynamical systems directly from measurement data [12]. From an adaptive control perspective, data-driven optimal control methods have been proposed to improve system performance by continuously updating control parameters under uncertain conditions [13]. Furthermore, data-driven reconstruction techniques have demonstrated their capability for identifying and recovering the underlying dynamics of complex large-scale systems [14]. Finally, Sparse Identification of Nonlinear Dynamics (SINDy) provides an alternative framework for discovering governing equations directly from observed data by exploiting sparse representations of nonlinear dynamics [15]. These developments provide the foundation for the proposed framework, which integrates data-driven modeling and advanced control strategies for robust nonlinear system regulation.

2. Contributions of This Research

To address the aforementioned challenges, the primary contributions of this paper are summarized as follows:

1. A Novel Data-Driven Identification Framework: We introduce a robust identification scheme that integrates Dynamic Mode Decomposition with Control (DMDc) with a rigorous spectral analysis procedure to quantify model fidelity and isolate dominant system dynamics.
2. Robust Optimal Control Synthesis: We propose an optimal sliding mode control strategy derived from the discrete Riccati equation, which explicitly incorporates actuator constraints and signal rate limits to handle practical operational restrictions.
3. Rigorous Stability and Safety Verification: A significant aspect of this study is the theoretical foundation of the controller. Specifically, Lyapunov-based stability analysis is utilized to prove the global convergence and robustness of the proposed control law, ensuring stable performance under uncertain conditions and unmodeled dynamics

To address the aforementioned challenges, the main contributions of this paper are summarized as follows. First, a novel framework for data-driven modeling via Dynamic Mode Decomposition with Control (DMDc) is developed to identify the system dynamics directly from input-output data. Second, the identified model is evaluated through spectral analysis, including eigenvalue and singular value investigations, in order to capture the dominant modes and validate the effectiveness of the extracted dynamic structure. Third, an optimal sliding mode control law is constructed based on the identified model, where the control gains are obtained by solving a cost-based optimization problem together with the discrete Riccati equation while explicitly considering control effort and rate constraints. Finally, Lyapunov-based stability analysis is employed to verify the stability, robustness, and convergence properties of the proposed control framework under system uncertainties.

3. System Formulation

Data-Driven Modeling via Dynamic Mode Decomposition The nonlinear system under consideration is described in discrete time as where $x_k \in \{R\}^n$denotes the state vector,$u_k \in \{R\}^m$represents the control input vector, and $f(.)$ is the unknown nonlinear system mapping. Since obtaining an accurate analytical model for many complex nonlinear systems is often difficult, the main objective of this section is to extract an effective local linear model directly from measured data. Such a model is expected to approximate the system behavior with acceptable accuracy over a finite operating interval. Accordingly, the local data-driven model is expressed as

$$x_{k+1} \approx Ax_k + Bu_k$$

where $A \in \{R\}^{n\times n}$ and $B \in \{R\}^{n\times m}$ are the unknown system matrices to be estimated from data. The identification of A and B is performed solely on the basis of measured input-output or state-input data, without requiring an explicit physics-based model of the system.

3.1. Sliding-Window Dynamic Mode Decomposition with Control

To enhance the adaptability of the identified model to time-varying dynamics, parametric uncertainties, and nonlinear system behavior, the identification process is carried out over sliding time windows. Within each window, a set of state and input data is collected, and a local model corresponding to that specific time interval is extracted. As the window moves forward in time, the system matrices A and B are updated recursively, allowing the identified model to track the system behavior under varying operating conditions. This strategy is particularly important for systems subject to rapid parameter variations, time-varying disturbances, or strong nonlinearities, since the resulting DMDc model is not treated as a fixed global representation. Instead, it acts as a local, data-driven, and continuously updated model that is refined as new measurements become available.

3.2. Data Organization

Data collected over a time window containing N samples are organized into state and input matrices. The state snapshot matrix and its one-step-ahead shifted counterpart are defined as:

$$X = \begin{pmatrix} x_1 & x_2 & \cdots & x_{N-1} \end{pmatrix}$$

$$X^+ = \begin{pmatrix} x_2 & x_3 & \cdots & x_N \end{pmatrix}$$

and the corresponding input matrix is given by

$$X^+ = AX + B$$

To estimate the system matrices A and B simultaneously, the augmented data matrix is introduced as

$$\Omega = \begin{bmatrix} X \\ Y \end{bmatrix} \in R^{((n+m)\times(N-1))}$$

and the parameter matrix is defined by

$$G = \begin{bmatrix} A & B \end{bmatrix} \in R^{(n\times(n+m))}$$

Using these definitions, the identification problem can be written in the compact form

$$X^+ \approx G\Omega$$

Based on this formulation, the simultaneous estimation of A and B is cast as a least-squares problem.

Theorem 3.1. Use `\begin{}...\end{}` for all Definitions, Lemmas, Theorems.

$$a^2 + b^2 = c^2 \tag{3.1}$$

$$G^\star = \arg\min_G \left\| X^+ - G\Omega \right\|_F^2$$

Accordingly, the simultaneous estimation of the system matrices A and B is formulated as the following least-squares optimization problem:

$$G^\star = X^{+\dagger}$$

where Ω denotes the Moore-Penrose pseudoinverse of the matrix. To mitigate the effect of measurement noise and retain the dominant dynamical components, singular value decomposition (SVD) is applied to the data matrix. This step enables a reduced-order representation of the system while preserving its most significant dynamical features.
At the end of this stage, the state-space model of the system is fully identified from the measured data and subsequently employed for controller design. As a result, the identified model can be expressed in the following form:

$$G = X^+ V \Sigma^{-1} U^T$$

It will be. Finally, the system matrices are extracted as:

$$A = X^+ V \Sigma^{-1} U_1^T$$

$$B = X^+ V \Sigma^{-1} U_2^T$$

$$\Omega = \tilde{U}\tilde{\Sigma}\tilde{V}^\star$$

4. Validation of the Identified Model

After estimating the dynamic matrices Aˆ Bˆ using the Dynamic Mode Decomposition with control (DMDc) method, the predicted model for the data within the considered time window is defined as follows:

$$X^+ = \hat{A}X + \hat{B}U$$

where $\hat{X}^+$ denotes the matrix of reconstructed or predicted states generated by the identified model. To ensure that the extracted model is suitable for controller design, its level of agreement with the actual data must be evaluated. One of the main criteria for this purpose is the model reconstruction error, which is defined as follows:

$$\eta_m = \frac{\left\| X^+ - \hat{X}^+ \right\|_F}{\|X^+\|_F}$$

where $\| \, \|_F$ denotes the Frobenius norm, and η_m is a dimensionless index used to evaluate the relative accuracy of the identified model. If the value of this index in the ith time window is smaller than a prescribed acceptable bound, that is, $\eta_{\{m\}}^{\{(i)\}} \leqslant \eta_{\max}$, then the identified model in that window is considered valid and can be used for the design or update of the control law.
To optimize the controller performance, the discrete-time cost function is defined as follows:

$$J = \sum_{k=0}^{\infty} \left(x_k^T Q x_k + u_k^T R u_k \right)$$

It is defined as follows. To achieve a suitable balance between tracking accuracy and control effort, a quadratic cost function is considered. Then, the discrete-time algebraic Riccati equation is solved as follows:

$$A^T P A - P - A^T P B \left(R + B^T P B \right)^{-1} B^T P A + Q = 0$$

By solving the discrete-time Riccati equation, the optimal correction to the control gain is computed, thereby improving the closed-loop performance in terms of stability and transient response. Accordingly, the corrective control gain is obtained as:

$$\Delta K = \left(R + B^{T}PB\right)^{-1} B^{T}PA$$

The final proposed control gain is given by

$$K = K_0 + \Delta K$$

where the final gain is obtained by combining the extracted spectral information with the optimal Riccati-based correction. This feature constitutes one of the main distinctions between the proposed method and conventional control design approaches. During the initial stages of system operation, the available input-output data are generally insufficient, and the snapshot matrices required for the DMDc algorithm are not yet fully constructed. As a result, the direct extraction of the optimal control gain is either not feasible or may lead to unstable behavior. For this reason, an initial stabilizing gain K_0 is incorporated into the control structure to guarantee the preliminary stability of the system. After sufficient data have been collected and the system dynamics have been gradually identified, an adaptive corrective term ΔK is computed based on the data-driven model and the solution of the discrete-time Riccati equation, and is then added to the initial gain. If you want a more explicit version, you can write:

$$u_k = -\left(K_0 + \Delta K\right) x_k$$

or, if δk is obtained from the Riccati-based update,

$$\Delta K = \left(R + B^{T}PB\right)^{-1} B^{T}PA$$

and therefore,

$$K = K_0 + \left(R + B^{T}PB\right)^{\{-1\}} B^{\{T\}}PA$$

$$K = K_0 + \left(R + B^{T}PB\right)^{-1} B^{T}PA$$

In this method, unlike the classical LQR design, the final control gain is formed by combining the initial gain derived from spectral analysis with the corrective term obtained from the Riccati equation.

5. Data-Driven Sliding Mode Controller Design

After determining the final control gain, the sliding surface is defined as follows:

$$s = Cx$$

It is defined as follows. The sliding surface is selected such that the desired system behavior on this surface is guaranteed. The equivalent control is given by:

$$u_{eq} = -(CB)^{-1}CAx$$

The equivalent control is responsible for maintaining the system motion on the sliding surface. The switching control is also defined as follows:

$$u_{sw} = -K\ \text{sign}(s)$$

The switching component is introduced to compensate for disturbances and uncertainties and to drive the system toward the sliding surface. Therefore, the final control law is given

$$u = -\left(CB\right)^{-1} CAx - K\ \text{sign}(s)$$

where is the control gain obtained from the previous section. Therefore, the final control law consists of an equivalent component and a robust component, and it makes use of the proposed gain extracted in the previous stage.

6. LMI-Based Stability Guarantee

For the augmented system, the control Lyapunov function (CLF) is defined as follows:

$$V(x_a) = x_a^T P x_a$$

To assess the stability of the closed-loop system, a quadratic control Lyapunov function is selected in the augmented state space. To guarantee closed-loop stability, it is required that:

$$A_a^T P A_a - P < 0$$

This condition must hold. Satisfying this inequality is necessary to ensure the monotonic decrease of the Lyapunov function and, consequently, the asymptotic stability of the system. To transform this nonlinear condition into a convex problem, the Schur complement lemma is employed, and the above condition is reformulated into the following LMI form:

$$\begin{bmatrix} A_a^T P A_a - P + M^T + M & M^T \\ M & -I \end{bmatrix} < 0$$

To eliminate this non linearity, a change of variables is performed:

$$M = KP$$

Satisfaction of this inequality guarantees the existence of a control Lyapunov function and establishes the stability of the closed-loop system.

Figure 1: Lyapunov-Constrained Data-Driven Control Algorithm

7. Simulation Result

A nonlinear Van der Pol oscillator is employed as a case study to demonstrate the performance of the developed control framework. The procedure involves extracting input-output trajectories, constructing the associated data matrices, and applying the proposed methodology for system identification, spectral analysis, and control synthesis. Simulation outcomes illustrate that the proposed scheme reliably reconstructs the system dynamics while achieving efficient control performance Dynamic Mode Decomposition (DMD) is employed to identify and reconstruct the dynamics of the nonlinear Van der Pol oscillator from sampled data. The results demonstrate that the extracted model is capable of reproducing the temporal behavior of the two state variables with high accuracy, where the reconstructed trajectories (red line) exhibit close agreement with the reference response (blue line). This underscores the capability of DMD in capturing a linear approximation of nonlinear dynamics while preserving the dominant system structure, particularly within the limit cycle region. However, the minor discrepancies observed highlight the method's inherent limitations in fully modeling strongly nonlinear behaviors and its sensitivity to the quality of the In this paper, the performance of an advanced control structure is investigated for non-periodic signal tracking. The controller design is established by integrating Lyapunov stability theory within the framework of Linear Matrix Inequalities (LMIs), subject to gain constraints based on the proposed method. Simultaneously,

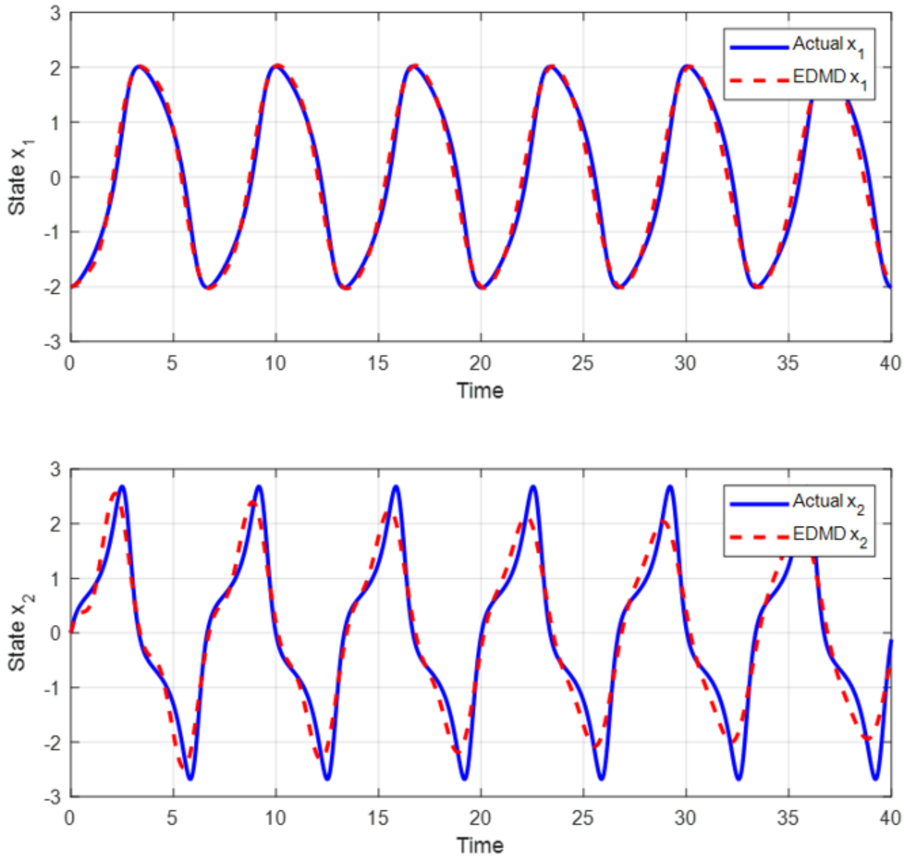

Figure 2: Comparison of the Data-Identified Model with the Reference Model In this study

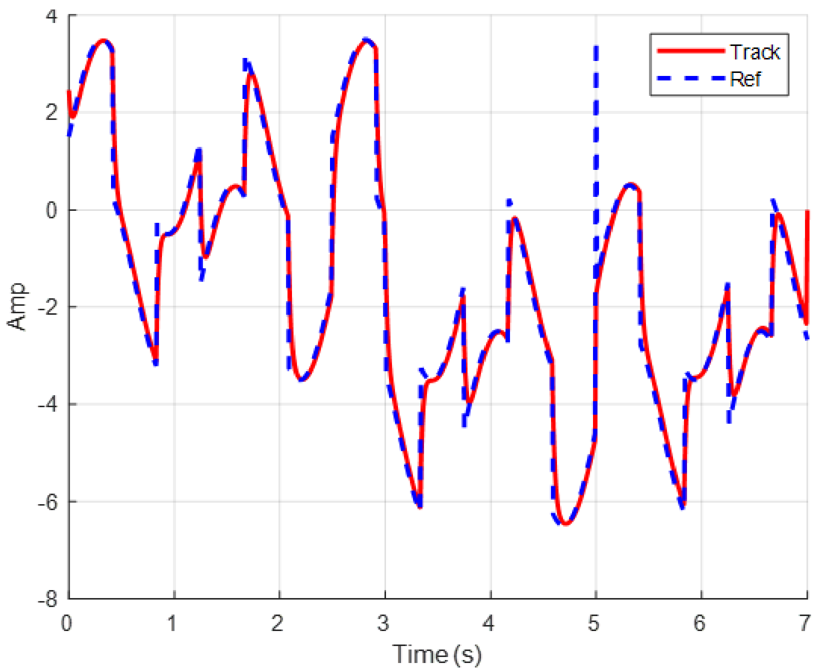

Figure 3: training-data-reference-signal-tracking

physical system constraints—including limitations on control effort and its derivative—are explicitly incorporated into the design process. The system demonstrates the ability to maintain stability and tracking performance under these prescribed constraints. This substantiates the effectiveness of the proposed framework in simultaneously ensuring stability, satisfying input constraints, and maintaining system safety in the presence of noise and external disturbances In this figure, the tracking error, defined as the difference

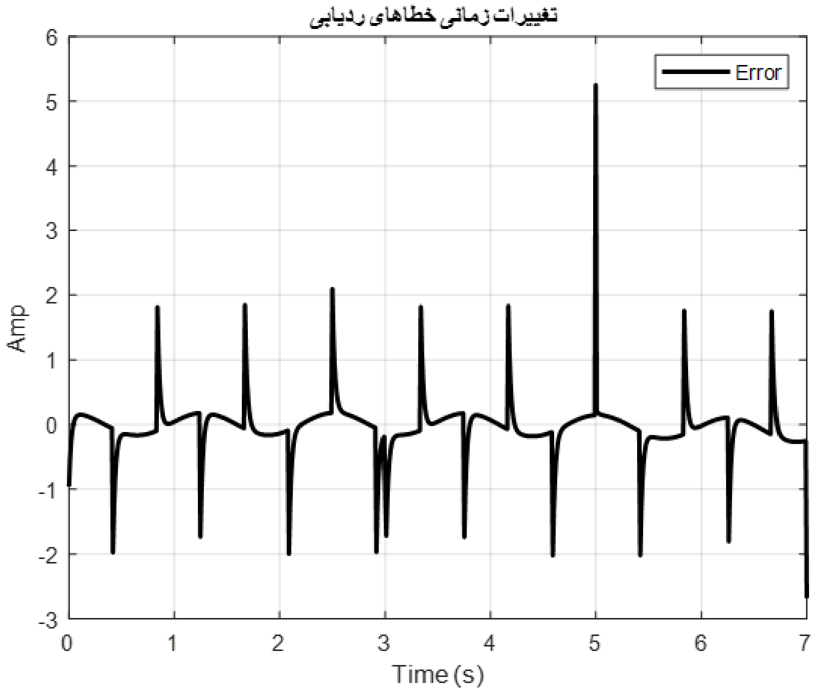

Figure 4: reference-signal-tracking-error

between the reference signal and the system output, is depicted under the proposed control scheme. As observed, the error fluctuates around zero for the majority of the time, indicating favorable convergence of the system to the reference signal. The transient peaks in the error are attributed to the rapid variations in the reference signal as well as the non-periodic nature of the system dynamics. At approximately t 5s, a noticeable increase in the tracking error occurs, which corresponds to the injection of an external disturbance. However, the error subsequently decays, returning to a near-zero range, thereby demonstrating the controller's capability to recover tracking performance in the presence of noise and disturbances. Furthermore, the boundedness of the error over time confirms the stability of the closed-loop system, validating the concurrent effectiveness of the Lyapunov constraints, LMI formulations, and Control Barrier Functions (CBF) in maintaining safe and stable system operation." In this figure, the sliding surface trajectory is illustrated within the Sliding Mode Control (SMC) framework, implemented via a data-driven structure. As observed, the sliding surface exhibits oscillatory behavior characterized by sharp transients. This reflects the inherent nature of SMC, which utilizes discontinuous control actions to force the system dynamics to converge onto the sliding surface. The sharp spikes observed in the sliding surface signal indicate the activation moments of the switching control, reflecting the system's effort to compensate for uncertainties and external disturbances. Nevertheless, the boundedness of the trajectory and its tendency to oscillate around zero demonstrate that both the reaching condition and the subsequent sliding motion are effectively satisfied by the controller design. Within the data-driven framework, this behavior signifies that the sliding surface is tuned—solely based on identified data rather than relying on an exact analytical model—such that the stability and convergence of the closed-loop system are rigorously guaranteed.

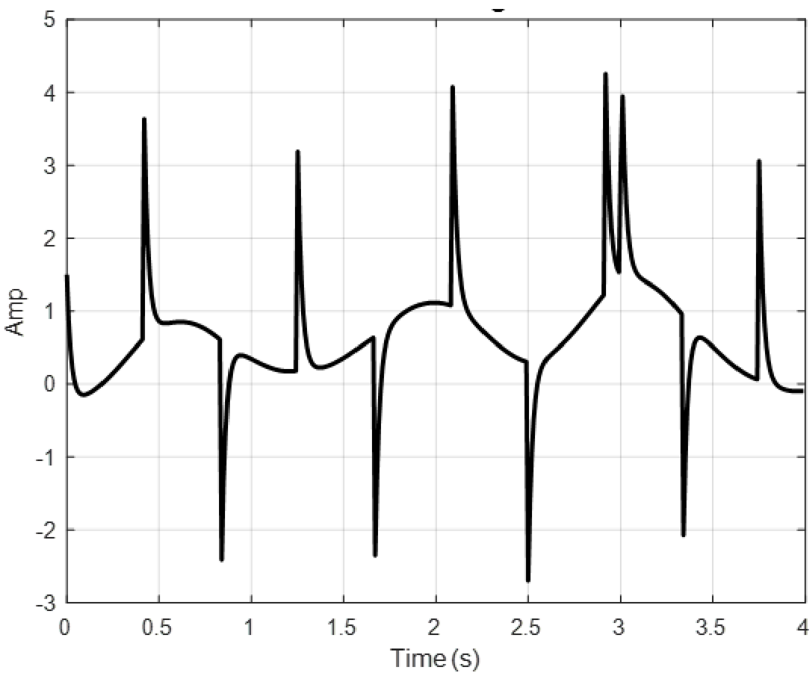

Figure 5: Sliding surface variations

8. Conclusion

“In this study, a robust data-driven control framework is proposed, in which a local linear model of the system is first extracted from input-output data using Dynamic Mode Decomposition (DMD), and then refined through model reduction to obtain the dominant dynamics with reduced noise contamination. The validity of the identified model is assessed through spectral analysis of the eigenvalues and singular values. To address the issue of insufficient data during the initial time instants, an initial stabilizing gain, K_0, is determined based on spectral information. Once sufficient data become available, the gain correction is computed in the form of K by solving the discrete Riccati equation, leading to the final gain $K = K_0 + K$. The final control law is designed by combining K with sliding mode control, while the closed-loop stability is examined within the CLF/Lyapunov framework under LMI conditions derived via the Schur complement. In addition, the control effort and its rate of variation are incorporated into the design, together with state bounds and model deviation, so that safety and robust stability are simultaneously guaranteed

References

[1] H. K. Khalil, Nonlinear Systems, 3rd ed., Prentice Hall, Upper Saddle River, NJ, (2002). 1

[2] S. L. Brunton and J. N. Kutz, Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control, Cambridge University Press, Cambridge, (2019). 1

[3] J. N. Kutz, S. L. Brunton, B. W. Brunton and J. L. Proctor, Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems, Society for Industrial and Applied Mathematics, Philadelphia, PA, (2016). 1

[4] J. L. Proctor, S. L. Brunton and J. N. Kutz, Dynamic mode decomposition with control, SIAM J. Appl. Dyn. Syst., 15 (2016), 142–161. 1

[5] V. Utkin, J. Guldner and J. Shi, Sliding Mode Control in Electromechanical Systems, 2nd ed., CRC Press, Boca Raton, FL, (2009). 1

[6] Z. S. Hou and Z. Wang, From model-based control to model-free control: A survey, IEEE Trans. Autom. Control, 58 (2013), 2190–2204. 1

[7] A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath and P. Tabuada, Control barrier functions: Theory and applications, IEEE Control Syst. Mag., 39 (2019), 119–137. 1

[8] A. Levant, Sliding mode control: Mathematical tools, design and applications, IEEE Trans. Autom. Control, 57 (2012), 3178–3192. 1

[9] B. D. O. Anderson and J. B. Moore, Optimal Control: Linear Quadratic Methods, Prentice-Hall, Englewood Cliffs, NJ, (1990). 1

[10] I. Mezić, Analysis of fluid flows via spectral properties of the Koopman operator, Annu. Rev. Fluid Mech., 45 (2013), 357–378. 1

[11] M. Korda and I. Mezić, Linear predictors for nonlinear dynamical systems: Koopman operator meets model predictive control, Automatica, 93 (2018), 149–160. 1

[12] T. Sahai, A. Muralidharan and J. N. Kutz, Data-driven controller design for non-linear systems using dynamic mode decomposition, Control Theory Technol., 14 (2016), 316–324. 1

[13] H. Liu, C. Wang and F. Lewis, Data-driven adaptive optimal control of nonlinear systems, IEEE Trans. Neural Netw. Learn. Syst., 28 (2017), 2933–2945. 1

[14] K. Zhang and I. A. Hiskens, Data-driven reconstruction of the Koopman operator: Application to power systems, IEEE Trans. Power Syst., 31 (2016), 3113–3125. 1

[15] S. L. Brunton, J. L. Proctor and J. N. Kutz, Discovering governing equations from data by sparse identification of nonlinear dynamics, Proc. Natl. Acad. Sci. USA, 113 (2016), 3932–3937. 1